Curvature

aMATHing day

22 April 2015

THE UNIVERSITY
of ADELAIDE

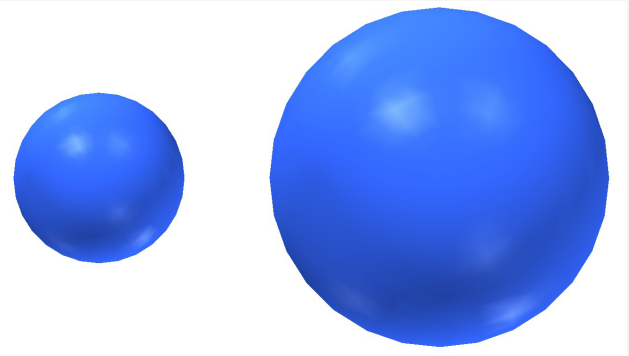
The toolkit of geometry

Things we can measure:

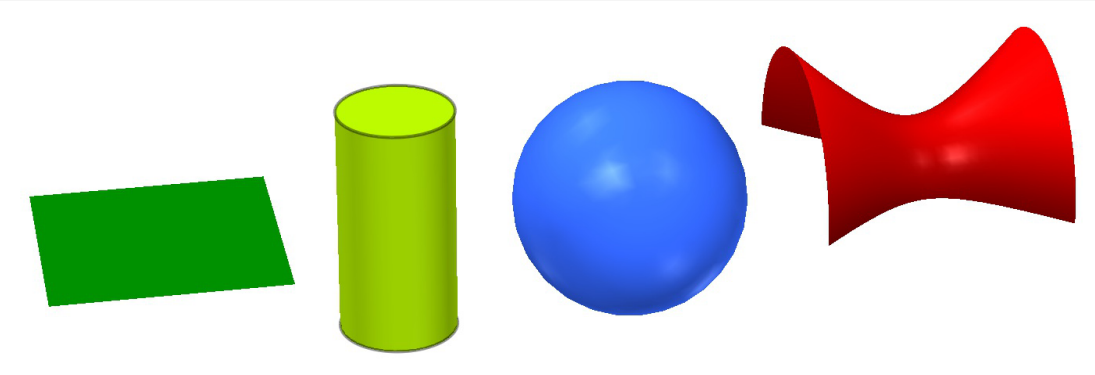
- Distance
- Length/Area/Volume
- Angles

What determines the geometry of an object?

Size/distance

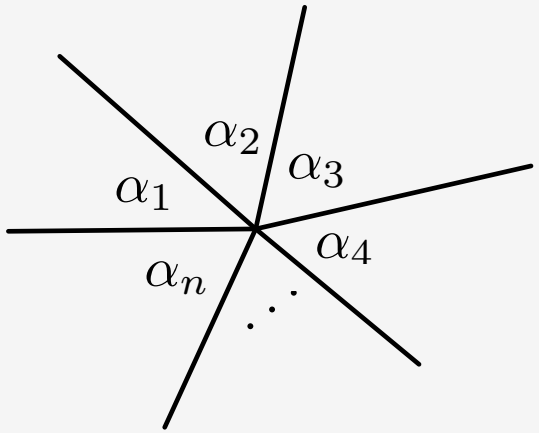


Curvature

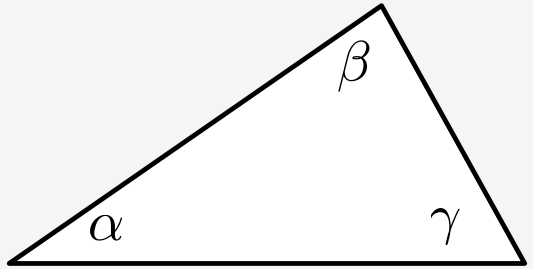


Geometry in the (flat) plane

$\alpha_1 + \alpha_2 + \ldots + \alpha_n = 360°$



$\alpha + \beta + \gamma = 180°$



Which curvature?

- Cylinders are curved.
- Unroll a cylinder to the plane:
  Lenghts, angles do not change.

$\rightsquigarrow$ plane geometry = cylinder geometry

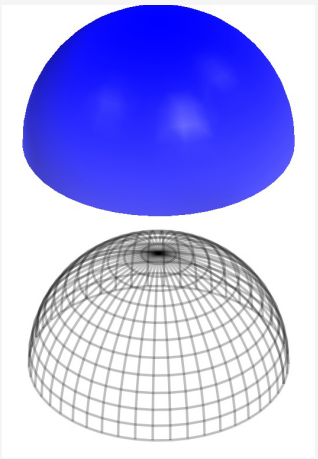
Curvature should express how geometry on an object fails to be the plane geometry.
We call this curvature Gauss curvature.

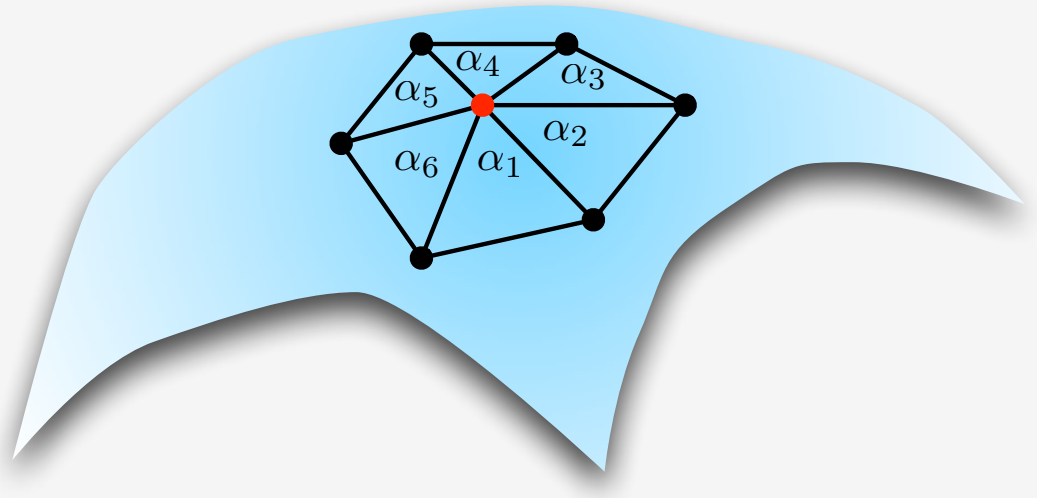
Named after Carl Friedrich Gauss
(1777–1855)

Angle sums

Approximate curved surface by polygons:

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n \neq 360^\circ$$





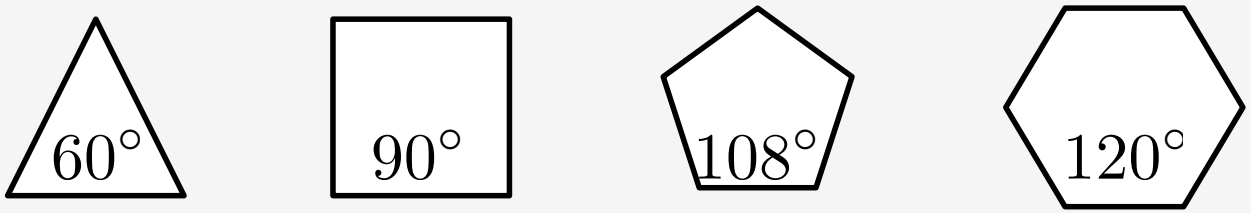
$360^\circ > \alpha_1 + \alpha_2 + \ldots + \alpha_n$: positive curvature.

$360^\circ < \alpha_1 + \alpha_2 + \ldots + \alpha_n$: negative curvature.

$360^\circ = \alpha_1 + \alpha_2 + \ldots + \alpha_n$: zero curvature (flat).

– Activity –

Craft your own curved surface by gluing polygons.

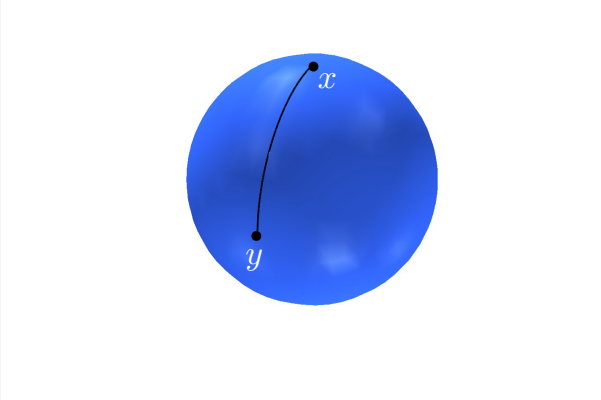


– Activity –

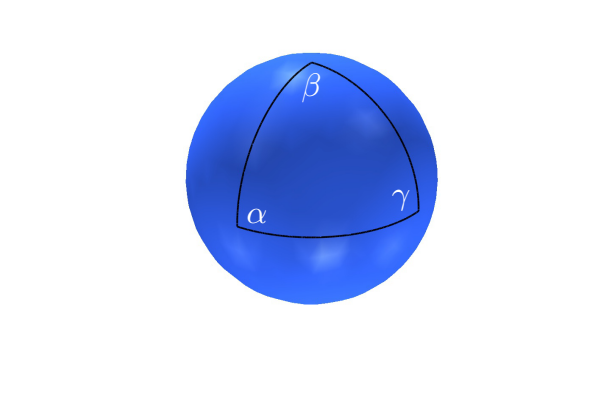
How does the sum of the angles in a triangle on the sphere differ from a plane triangle?

"Straight line" from x to y:

- Piece of string connecting x and y on the sphere.



- Triangle on the sphere:



– Activity –

Maps vs. globes:

- or: Flat vs. curved.
- Compare different world maps to a globe.
- Do the maps give a precise measurement of the earth's proportions?

Gauss' Remarkable Theorem

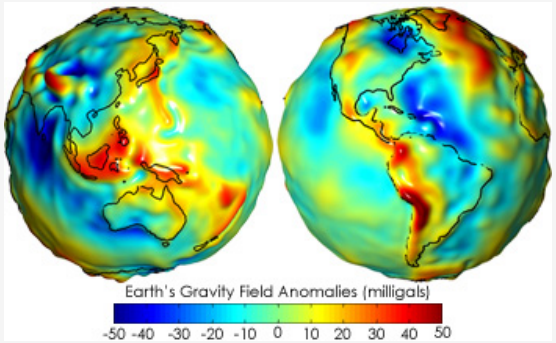
THEOREMA. *Si superficies curva in quamcunque aliam superficiem explicatur, mensura curvaturae in singulis punctis invariata manet.*

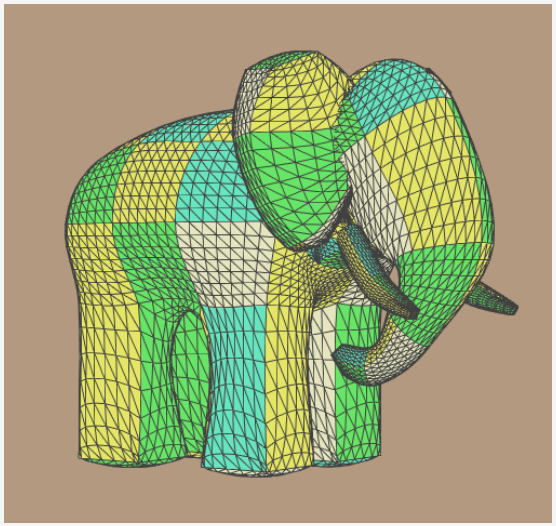
If a curved surface is developed upon any other surface, the measure of curvature in each point remains unchanged.

- "develop" = deform one surface into an other, without stretching or tearing it
- Map: zero curvature. Globe: positive curvature.
  $\Rightarrow$ no map of any kind can display the world without distortion.

Why curvature?

Geodesy – measuring the surface of the earth





Computer aided geometric design – creating free-form surfaces for computer graphics, robotics and construction

General relativity – mathematical formalism of gravitation resembles curvature in spacetime